Proceedings of the
International Conference on Mechanical Engineering 2011
(ICME2011) 18-20 December 2011, Dhaka, Bangladesh

ICME11-TH-022

EFFECTS OF PRESSURE WORK AND RADIATION ON MAGNETOHYDRODYNAMIC FREE CONVECTION FLOW ALONG A SPHERE WITH JOULE HEATING

M. Miraj[1], M. A. Alim[2] and L. S. Andallah[3]

[1]Department of Mathematics, Dhaka Commerce College, Bangladesh
[2]Department of Mathematics, BUET, Bangladesh
[3]Department of Mathematics, Jahangirnagar University, Savar, Dhaka, Bangladesh

ABSTRACT

The effects of pressure work and radiation on magnetohydrodynamic (MHD) free convection flow along a sphere with joule heating have been studied in this paper. The governing boundary layer equations with associated boundary conditions are converted to non-dimensional form using the appropriate transformations. The resulting nonlinear systems of partial differential equations are mapped into the domain along a sphere and then solved numerically using implicit finite difference method, known as Keller-box scheme. The solutions are expressed in terms of the skin friction coefficient, the rate of heat transfer, the velocity profiles and temperature profiles over the whole boundary layer. The effects of varying radiation parameter *Rd*, pressure work parameter *Ge* , magnetic parameter *M*, joule heating parameter *J* and the Prandtl number *Pr* are shown graphically and discussed. The effects of pressure work and radiation on flow and temperature fields have been found significant.

Keywords: Natural Convection, Thermal Radiation, Prandtl Number, Pressure Work, Nusselt Number, Joule Heating and Magnetohydrodynamics.

1. INTRODUCTION

The effects of pressure work and radiation on MHD free convection flow along various geometrical shapes such as vertical flat plate, cylinder, sphere etc, have been studied by many investigators. Radiation effects on free convection flow are important in the context of space technology and processes involving high temperatures but comparatively less information about the effects of radiation on the boundary layer flow is available than convection and conduction heat transfer from fluid flow past a body. Hossain and Takhar [1] have analyzed the effects of radiation using the Rosseland diffusion approximation. Alim et al. [2-3] consider the pressure work effect along a circular cone and stress work effects on MHD natural convection flow along a sphere and Akhter [4] studied the effects of pressure work on natural convection flow around a sphere with radiation heat loss. Limitations of this approximation are discussed briefly in Özisik [5]. Miraj et al. [6] studied the effect of radiation on natural convection flow on a sphere in presence of heat generation. Molla et al [7] have studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation or absorption. Hossain [8] introduced the viscous and Joule heating effects on MHD-free convection flow with variable plate temperature. In the present work, the effects of pressure work and radiation on magnetohydrodynamic free convection flow along a sphere with joule heating have been investigated. Transformed governing equations of the present problem with the appropriate boundary conditions have been solved numerically using Keller box method by Keller [10].

2. FORMULATION OF THE PROBLEM

It is assumed that the constant temperature at the surface of the sphere is T_w, where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity and (U,V) are velocity components along the (X,Y) axes. The physical configuration considered is as shown in Figure 1.

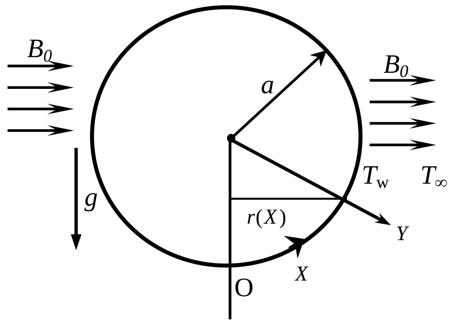


Fig 1. Physical model and coordinate system

Under the Boussinesq and boundary layer approximations, the governing equations for continuity, momentum and energy take the following forms:

$$\frac{\partial}{\partial X}(rU)+\frac{\partial}{\partial Y}(rV)=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2}+g\beta(T-T_\infty)\sin\left(\frac{X}{a}\right)-\frac{\sigma_0 B_0{}^2}{\rho}U \tag{2}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{k}{\rho\, c_p}\left(\frac{\partial^2 T}{\partial Y^2}-\frac{1}{k}\frac{\partial q_r}{\partial Y}\right)+\frac{T\beta}{\rho c_P}U\frac{\partial P}{\partial X}+\frac{\sigma_0 B_0{}^2}{\rho\, c_p}U^2 \tag{3}$$

With the boundary conditions

$$U=V=0,\ T=T_\infty\ \ at\ \ Y=0$$
$$U\to 0,\ T\to T_\infty\ \ as\ \ Y\to\infty \tag{4}$$

where $r(X)=a\sin(X/a)$ is the radial distance from the centre to the surface of the sphere, k is the thermal conductivity, β is the coefficient of thermal expansion, B_0 is the strength of magnetic field, σ_0 is the electrical conductivity, $\nu(=\mu/\rho)$ is the kinematic viscosity, μ is the viscosity of the fluid, ρ is the density and. c_p is the specific heat due to constant pressure. The above equations are non-dimensionalised using the following new variables:

$$\xi=X/a,\quad \eta=YGr^{1/4}/a,$$
$$u=aUGr^{-1/2}/\nu,\ \ v=aVGr^{-1/4}/\nu \tag{5}$$

$$\theta=(T-T_\infty)/(T_w-T_\infty),$$
$$Gr=g\beta(T_w-T_\infty)a^3/\nu^2,\quad \theta_w=T_w/T_\infty \tag{6}$$

where Gr is the Grashof number, θ is the non-dimensional temperature function, θ_w is the surface temperature parameter and q_r is the radiation heat flux. The Rosseland diffusion approximation proposed by Siegel and Howell [9] is given by simplified radiation heat flux term as:

$$q_r=-\frac{4\sigma}{3(a_r+\sigma_s)}\frac{\partial T^4}{\partial Y} \tag{7}$$

where a_r is the Rosseland mean absorption co-efficient, σ_s is the scattering co-efficient and σ is the Stefan-Boltzmann constant. Substituting (5) and (6) into Eqs. (1), (2) and (3) lead to the following non-dimensional equations

$$\frac{\partial}{\partial\xi}(ru)+\frac{\partial}{\partial\eta}(rv)=0 \tag{8}$$

$$u\frac{\partial u}{\partial\xi}+v\frac{\partial u}{\partial\eta}=\frac{\partial^2 u}{\partial\eta^2}+\theta\sin\xi-\frac{\sigma_0 B_0{}^2 a^2}{\rho\nu\ \ Gr^{\frac{1}{2}}}u \tag{9}$$

$$u\frac{\partial\theta}{\partial\xi}+v\frac{\partial\theta}{\partial\eta}=\frac{1}{Pr}\frac{\partial}{\partial\eta}[\{1+\frac{4}{3}Rd\left(1+(\theta_w-1)\theta\right)^3\}\times\frac{\partial\theta}{\partial\eta}]+Ge(\theta+\frac{T_\infty}{T_w-T_\infty})u+Ju^2 \tag{10}$$

where Ge is the pressure work parameter, J is the joule heating parameter, Pr is the Prandtl number and Rd is the radiation parameter defined respectively as

$$Ge=g\beta a/c_P,\ J=\sigma_0 B_0{}^2\nu/[\rho c_P(T_w-T_\infty)]$$
$$\Pr=\nu c_P/k\ \text{ and }\ Rd=4\sigma T_\infty^3/[k(a_r+\sigma_s)]$$

The boundary conditions (4) then reduce to

$$u=v=0,\ \theta=1\ \text{ at }\ \eta=0$$
$$u\to 0,\ \theta\to 0\ \text{ as }\ \eta\to\infty \tag{11}$$

To solve Eqs. (9) and (10) with the help of following variables

$$\psi=\xi r(\xi)f(\xi,\eta),\ \theta=\theta(\xi,\eta),\ r(\xi)=\sin\xi \tag{12}$$

where ψ is the stream function defined by

$$u=\frac{1}{r}\frac{\partial\psi}{\partial\eta},\ \ v=-\frac{1}{r}\frac{\partial\psi}{\partial\xi} \tag{13}$$

Using the above transformed values in Eqs. (9) and (10) and simplifying, we have the following equation:

$$\frac{\partial^3 f}{\partial\eta^3}+\left(1+\frac{1}{\sin\xi}\xi\cos\xi\right)f\frac{\partial^2 f}{\partial\eta^2}+\theta\frac{\sin\xi}{\xi}-\left(\frac{\partial f}{\partial\eta}\right)^2-M\frac{\partial f}{\partial\eta}=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta}-\xi\frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right) \tag{14}$$

$$\frac{1}{Pr}\frac{\partial}{\partial\eta}\left\{\left(1+\frac{4}{3}Rd(1+(\theta_w-1)\theta)^3\right)\frac{\partial\theta}{\partial\eta}\right\}+\left(1+\frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial\theta}{\partial\eta}+Ge\left(\theta+\frac{T_\infty}{T_w-T_\infty}\right)\xi f'+J\xi^2\left(\frac{\partial f}{\partial\eta}\right)^2=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\theta}{\partial\eta}\frac{\partial f}{\partial\xi}\right) \tag{15}$$

The boundary conditions are

$$f=f'=0,\ \theta=1\ \text{ at }\ \eta=0$$
$$f'\to 0,\ \theta\to 0\ \text{ as }\ \eta\to\infty \tag{16}$$

It can be seen that near the lower stagnation point of the sphere, i.e., $\xi \approx 0$, Eqs. (14) and (15) reduce to the following ordinary differential equations:

$$f''' + 2f\,f'' - f'^2 + \theta - Mf' = 0 \quad (17)$$

$$\frac{1}{Pr}\left[\left\{1+\frac{4}{3}Rd\left(1+(\theta_w-1)\theta\right)^3\right\}\theta'\right]' + 2f\theta' = 0 = 0 \quad (18)$$

Subject to the boundary conditions

$$f(0) = f'(0) = 0,\ \theta(0) = 1$$
$$f' \to 0,\ \theta \to 0 \ \text{as}\ \eta \to \infty \quad (19)$$

The skin friction coefficient C_f and Nusselt number Nu, which can be written in non- dimensional form as

$$C_f = a^2 Gr^{-3/4}\tau_w / \mu\nu$$
$$\text{and} \quad Nu = aGr^{-1/4} q_w / \left[k(T_w - T_\infty)\right] \quad (20)$$

where τ_w is the shear stress and q_w is the heat flux at the surface defined respectively as

$$\tau_w = \mu\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$$

and $q_w = (q_c)_{Y=0} + (q_r)_{Y=0} = -k\left(\frac{\partial T}{\partial Y}\right)_{Y=0} + (q_r)_{Y=0}$

where q_c is conduction heat flux and q_r is the radiation heat flux. Using Eqs. (5), (6), boundary condition (19) and putting the values of τ_w and q_w in Eq.(20) and simplifying we get the following equations

$$Nu = -\left(1+\frac{4}{3}Rd\,\theta_w^{\ 3}\right)\theta'(\xi,0)$$
$$\text{and}\quad C_f = \xi f''(\xi,0) \quad (21)$$

We discuss the velocity profiles as well as the temperature profiles for a selection of parameter sets.

3. RESULTS AND DISCUSSION

Numerical values of the present problem are obtained in terms of velocity profiles, temperature profiles, skin friction coefficient and the rate of heat transfer. The effects for different values of radiation parameter *Rd* the velocity profiles and temperature profiles in case of Pr = 0.72, M = 0.50, J = 0.50 and Ge = 1.00 are shown in Figures 2(a) and 2(b) respectively. It is observed from Figure 2(a) that the velocity as well as the boundary layer thickness increase with increasing radiation parameter *Rd*. Figure 2(b) displays the results for the higher values of radiation parameter *Rd*, the peak value of temperature is less and the thermal boundary layer thickness increases due to the lower temperature gradient. In Figure 3(a) it is shown that when the Prandtl number *Pr* increases the velocity and the boundary layer thickness decrease. It is observed from Figure 3(b) that for higher values of Prandtl number *Pr*, the maximum value of temperature is higher and the thermal boundary layer thickness decreases due to the higher temperature gradient. It has been seen from Figure 4(a) that the velocities rise up to the position of η = 1.23788, 1.30254, 1.36929, 1.43822 and 1.50946 for M = 0.10, 0.50, 1.00, 1.50 and 2.25 respectively and from those position of η velocities fall down. Figure 4(b) display results for the temperature profiles, for different values of magnetic parameter *M* with pertinent parameters. It has been seen from Figures 5(a) and 5(b) that as the joule heating parameter *J* increases, both the velocity and temperature profiles increase up to the certain position of η and from those position of η the velocity and temperature profiles changes and cross the sides and then decrease little bit slowly. The variation of pressure work parameter *Ge* on velocity profiles and temperature profiles while Rd =1.00, Pr = 0.72, M=0.50 and J = 0.50 are shown in Figures 6(a) and 6(b). We observed that due to the change of *Ge* from 0.10 to 2.00 the velocity rises up 106.13%. Again in Figure 6(b) small value of *Ge* (= 0.10) gives the typical temperature profiles which is maximum temperature at the wall then it gradually decrease along η-direction.

Figures 7(a) and 7(b) display results of the skin friction coefficient and the rate of heat transfer for different values of radiation parameter Rd with Pr = 0.72, M = 0.50, J = 0.50 and Ge = 1.00. From Figure 7(a) has been that as the radiation parameter Rd increases, the skin friction coefficient decreases. It has been seen from Figure 7(b) that as the radiation parameter Rd increases, the rate of heat transfer increases up to the some position of ξ and from that position the rate of heat transfer decreases. Figures 8(a) and 8(b) display results of the skin friction coefficient and the rate of heat transfer for different values of Prandtl number Pr with Rd = 1.00, M = 0.50, J = 0.50 and Ge= 1.00. The skin friction coefficient increases for increasing values of Prandtl number *Pr*. Again Prandtl number *Pr* increases the rate of heat transfer increases up to the some position of ξ and from that position of ξ the rate of heat transfer decreases.

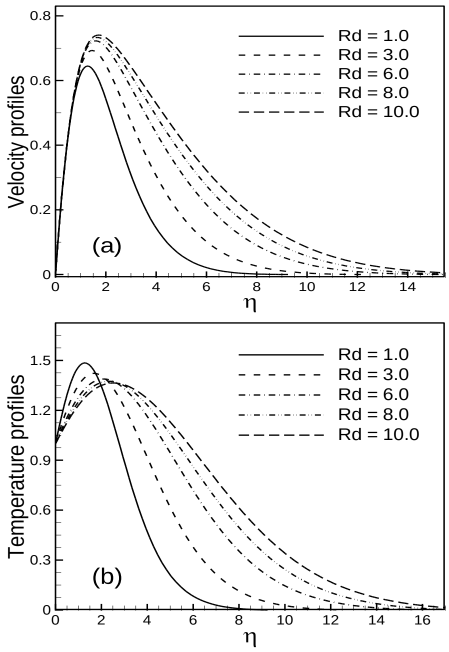


Fig 2. (a) Velocity profiles and (b) Temperature profiles for different values of *Rd* when Pr = 0.72, M = 0.50, J = 0.50 and Ge = 1.00

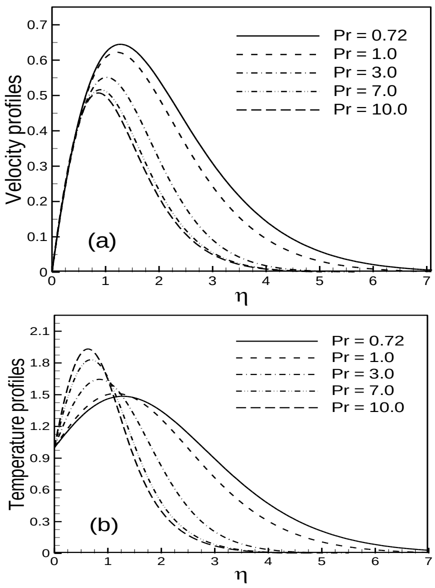


Fig 3. (a) Velocity profiles and (b) Temperature profiles for different values of *Pr* when *Rd* = 1.00, *M* = 0.50, *J* = 0.50 and *Ge* = 1.00

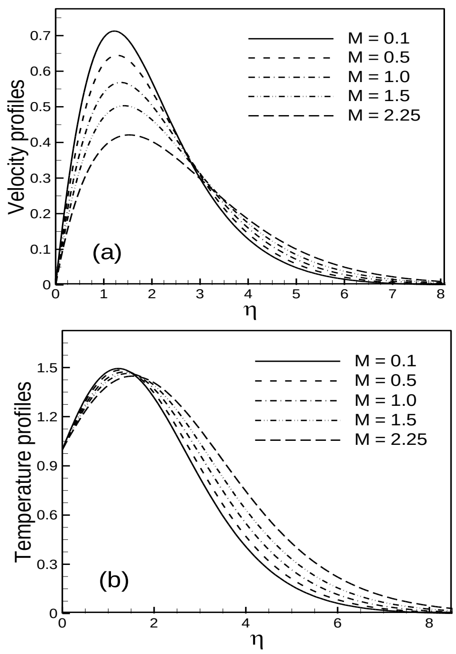


Fig 4. (a) Velocity profiles and (b) Temperature profiles for different values of *M* when *Rd* = 1.00, *Pr* = 0.72, *J* = 0.50 and *Ge* = 1.00

In Figure 9(a), the increasing values of magnetic parameter M while Rd =1.00, Pr = 0.72, J = 0.50 and Ge = 1.00, skin friction coefficient Cf decreases. The effects of rate of heat transfer for different values of magnetic parameter M with Rd =1.00, Pr = 0.72, J = 0.50 and Ge = 1.00 are shown in Figure 9(b). Here, it observed that at ξ = 0.03491, the rate of heat transfer decreases by 15.47% and at ξ = 1.03900, the rate of heat transfer increases by 16.53% for increasing of the magnetic parameter. From figures 10(a)-10(b) we observed that the skin friction coefficient Cf increases significantly and heat transfer coefficient *Nu* decreases for increasing values of joule heating parameter. Figure 11(a) shows the skin friction coefficient increases for different increasing values of pressure work parameter and in 11(b) gradually decreases for higher values of pressure work parameter

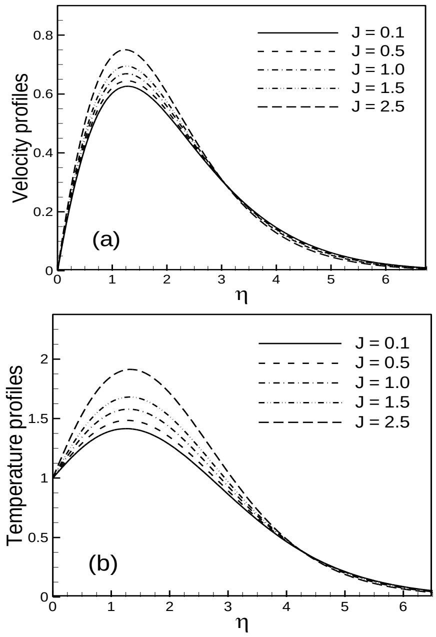


Fig 5. (a) Velocity profiles and (b) Temperature profiles for different values of *J* when *Rd* = 1.00, *Pr* = 0.72, *M* = 0.50 and *Ge* = 1.00

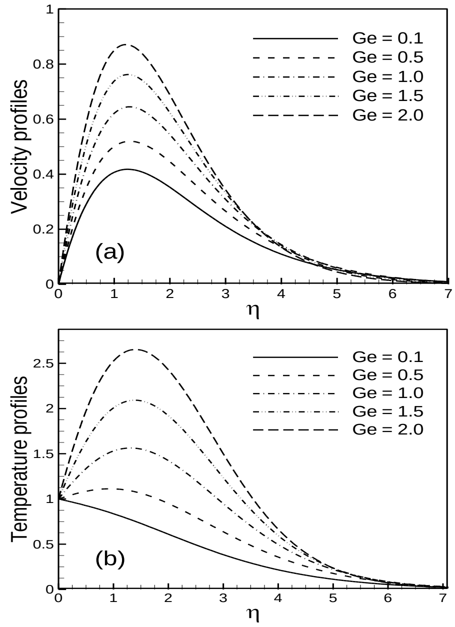


Fig 6. (a) Velocity profiles and (b) Temperature profiles for different values of *J* when *Rd* = 1.00, *Pr* = 0.72, *M* = 0.50 and *J* = 0.50

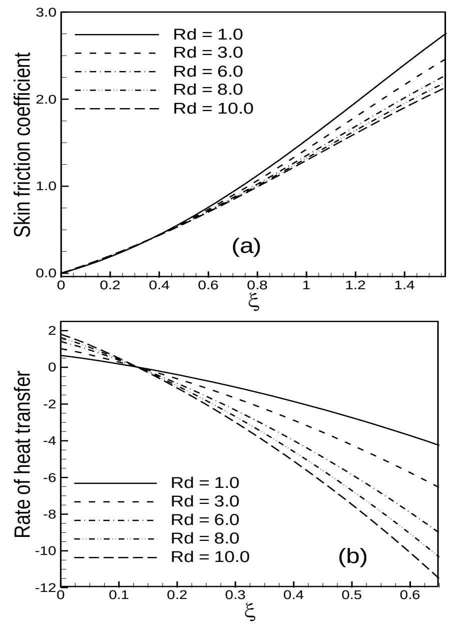


Fig 7. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *Rd* when *Pr* = 0.72, *M* = 0.50, *J* = 0.50 and *Ge* = 1.00

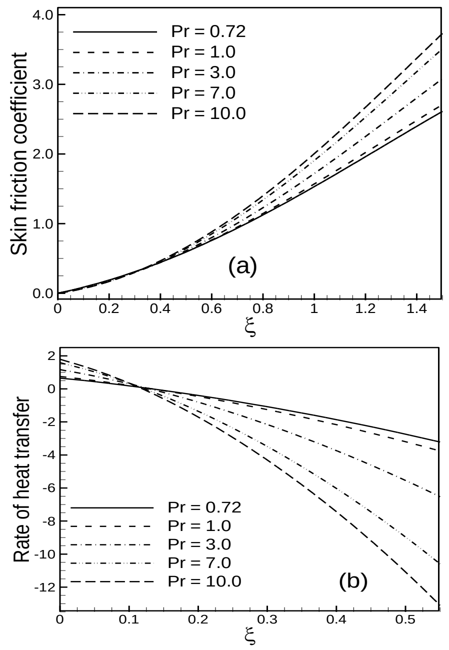
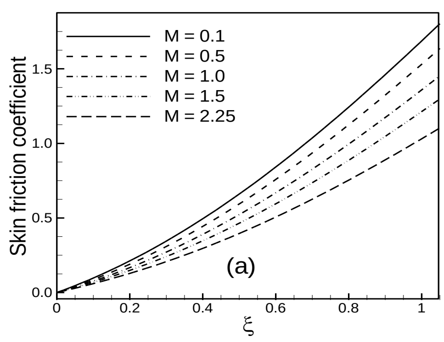
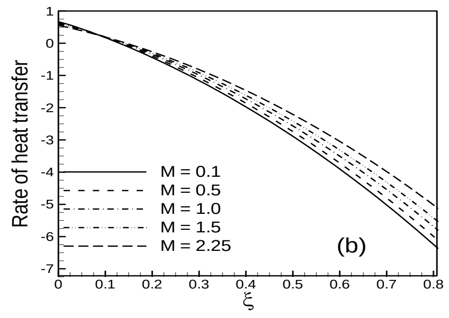


Fig 8. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *Pr* when *Rd* = 1.00, *M* = 0.50, *J* = 0.50 and *Ge* = 1.00



(a)



(b)

Fig 9. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *M* when *Rd* = 1.00, *Pr* = 0.72 , *J* = 0.50 and *Ge* = 1.00

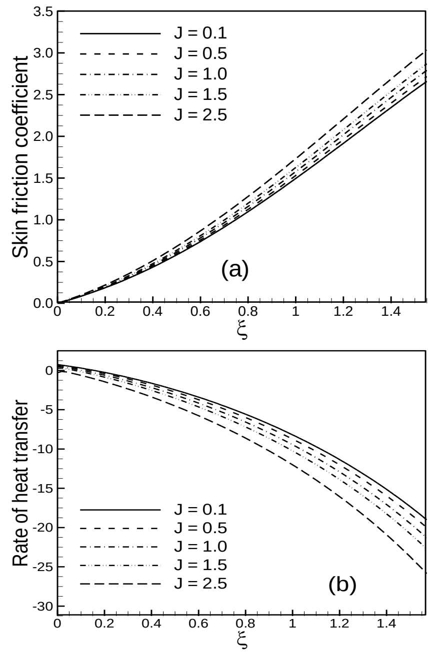


Fig 10. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *J* when *Rd* = 1.00, *Pr* = 0.72, *M* = 0.50 and *Ge* = 1.00

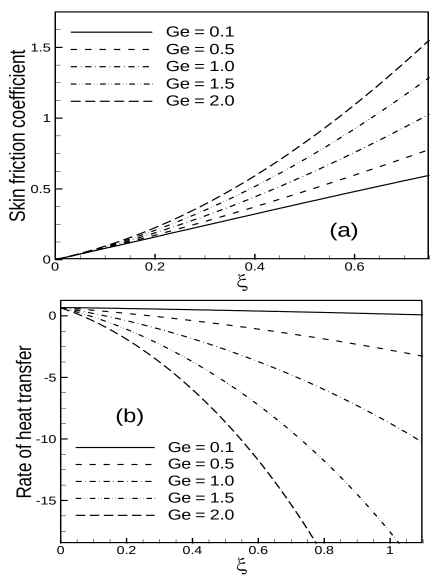


Fig 11. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *J* when *Rd* = 1.00, *Pr* = 0.72, *M* = 0.50 and *J* = 0.50

4. CONCLUSION

From the present investigation the following conclusions may be drawn:

The velocity profiles increases for increasing values of radiation parameter *Rd* , pressure work parameter *Ge* and the velocity profiles decreases for increasing values of Prandtl number *Pr.* Also the velocity profiles cross the sides for the values of magnetic parameter *M* and joule heating parameter *J.* The temperature profiles increases for increasing values of pressure work parameter *Ge.* But the temperature profiles cross the sides for the values of radiation parameter *Rd* , Prandtl number *Pr*, magnetic parameter *M* and joule heating parameter *J.* For increasing values of Prandtl number *Pr,* joule heating parameter *J* and pressure work parameter *Ge* the skin friction coefficient increases and for increasing values of radiation parameter *Rd* and magnetic parameter *M* and joule heating parameter *J* the skin friction coefficient decreases. The rate of heat transfer decreases significantly when the values of joule heating parameter *J* and pressure work parameter *Ge* increases.

5. REFERENCES

1. Hossain, M. A. and Takhar, H. S.,1996, "Radiation effect on mixed convection along a vertical plate with uniform surface temperature", Heat and Mass Transfer, 31: 243-248.
2. Alim, M. A., Alam, Md. M. and Chowdhury, Md. M. K., 2006, "Pressure work effect on natural convection flow from a vertical circular cone with suction and non-uniform surface temperature, Journal of Mechanical Engineering", The Institution of Engineers, Bangladesh, 36: 6-11.
3. Alim, M. A., Alam, Md. M. and Chowdhury, Md. M. K. , 2008, "Work Stress Effects on MHD Natural Convection Flow Along a Sphere", Thammasat International Journal of Science and Technology, 13(1):1-10.
4. Akhter, T. and Alim, M.A., 2010, "Effects of pressure work on natural convection flow around a sphere with radiation heat loss", Nonlinear Analysis: Modelling and Control, 15(3):287–298.
5. Özisik, M. N., 1973, *Radiative Transfer and Interactions with Conduction and Convection*, Wiley, New York, USA.
6. Miraj, M., Alim, M. A. and Mamun, M. A. H., 2010, "Effect of Radiation on Natural Convection Flow on a Sphere in Presence of Heat Generation", International communications in heat and mass transfer, 37(6):660-665.
7. Molla, Md. M., Taher, M. A., Chowdhury, Md. M. K. and Hossain, Md. A., 2005, "Magnetohydrodynamic Natural Convection Flow on a Sphere in Presence of Heat Generation", Nonlinear Analysis: Modelling and Control, 10(4):349–363.
8. Hossain, M. A., 1992, "Viscous and joule heating effects on MHD-free convection flow with variable plate temperature", International Journal of Heat and Mass Transfer, 35(12):3485-3487.
9. Siegel, R. and Howell, J. R., 1972, *Thermal Radiation Heat Transfer*, McGraw-Hill, New York, USA.
10. Keller, H. B., 1978, "Numerical methods in boundary layer theory", Annual Review of Fluid Mechanics, 10:417-433.

6. NOMENCLATURE

Symbol	Meaning	Unit
a	Radius of the sphere	(m)
a_r	Rosseland mean absorption coefficient	(m^3/s)
C_f	Skin-friction coefficient	–
Ge	Pressure work parameter	–
J	Joule heating parameter	–
M	Magnetic parameter	–
Nu	Nusselt number	
Pr	Prandtl number	–
T	Temperature of the fluid in the boundary layer	(K)
U	Velocity component along the surface	(ms^{-1})
V	Velocity component normal to the surface	(ms^{-1})
u	Dimensionless velocity along the surface	–
v	Dimensionless velocity normal to the surface	–
X	Coordinate along the surface	(m)
Y	Coordinate normal to the surface	(m)
ξ	Dimensionless coordinate along the surface	–
η	Dimensionless coordinate normal to the surface	–
ψ	Stream function	(m^2s^{-1})

7. MAILING ADRESS

M. A. Alim
Department of Mathematics,
Bangladesh University of Engineering and Technology,
Dhaka-1000, Bangladesh.
Email: maalim@math.buet.ac.bd